

## Refinements to the Neher-McGrath Model for Calculating the Ampacity of Underground Cables

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*Abstract* - The Neher-McGrath method has been widely accepted as an accurate and relatively simple way to calculate the ampacity of underground cables. It is based on a number of assumptions that simplify the mathematics, but at the same time limit the accuracy and scope of the model. Furthermore, the model relies upon correlations that are now dated, because more up-to-date and accurate heat transfer correlations are now available.

This paper examines improvements to the Neher-McGrath model in three different areas: a more accurate expression for the mutual heating parameter that accounts for unequal heating among the cables; improved correlations for the thermal resistance of a fluid layer that exists in pipe-type cables and cables installed in ducts; and a more accurate model for the thermal resistance of concrete duct banks with non-square cross-sections.

Example installations are then considered to illustrate how the incorporation of these changes will affect the ampacity of the installation. The refinements suggested result in a more complex mathematical algorithm and require more computational time, but the changes can result in significantly different ampacity values than the ones provided by the Neher-McGrath model.

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### I. INTRODUCTION

The first models proposed for calculating the current carrying capacity of underground cables date back to the late 1800's and early 1900's [1, 2]. The early ampacity models, as they are now known, remained rather crude and highly simplified until Neher and McGrath [3] published a comprehensive paper that, for the first time, considered all relevant factors that influence the temperature of modern cable installations. Since its publication in 1957, the Neher-McGrath technique has been widely accepted as the standard for determining the current-carrying capacity of underground cable installations.

The mathematical model outlined in the Neher-McGrath paper is based on numerous assumptions, some of which were necessitated by the computing capabilities of the 1950's. The heat transfer theory and correlations that are included in the model were the best available at that time. However, more accurate results are available today and these more recent developments should be incorporated into an upgraded and improved version of an underground ampacity model. The goals of this paper are to objectively examine several improvements to the Neher-McGrath model, to incorporate recent heat transfer correlations, and to remove several assumptions considered in the Neher-McGrath paper that limit the accuracy of the calculations.

While the suggested changes result in a more complex ampacity model, they can be easily incorporated into the mathematics without a significant increase in effort, particularly considering the computational capabilities of today's personal computers. The factors considered in this paper will not only improve the accuracy of the existing ampacity model, but will also extend the application of the model so that it will apply to a larger foundation of geometries and installation practices. In some cases the changes to the model suggest an increase in the existing ampacity values and in others, the changes will reduce the ampacity.

The refinements suggested in this paper have been included in an ampacity program called TOAD (Trench Optimization and Design) that was written as part of an EPRI (Electric Power Research Institute) contract. The TOAD program has a wide range of capabilities, including the ability to calculate the ampacity of cables surrounded by a backfill material. By varying the backfill resistivity and the amount of the backfill used around the cables, an engineer can quickly optimize the installation so that it achieves a maximum amount of power delivered for a fixed installation cost. The TOAD program is part of EPRI's Underground Transmission Workstation software package.

## II. MUTUAL HEATING FACTOR

The first improvement to the existing ampacity model involves the calculation of the mutual heating factor. Since the ampacity model assumes a one-dimensional heat transfer problem in which the temperature in the cable and surrounding environment varies only with radial distance from the cable, two-dimensional effects that exist must be handled in an approximate fashion. Therefore when the installation consists of more than one cable and two-dimensional heat transfer occurs in the earth portion of the circuit, the resistance of the earth must be modified to account for the temperature rise at one cable resulting from heat generated in all other cables in the installation. This effect, which is usually referred to as mutual heating, is incorporated into the factor  $F$ , which occurs in the expression for the earth resistance term ((44) in [3]).

$$R_e = 0.366 \rho_e m' \left[ \log_{10} \left( \frac{D_x}{d_e} \right) + (LF) \log_{10} \left( \left( \frac{4L}{D_x} \right) F \right) \right] \quad (1)$$

By utilizing the principle of superposition, and by restricting short-circuited shield operation to three-conductor cables and three, single-conductor cables in a duct, therefore equal heat generation in each cable, Neher-McGrath show that the mutual heating factor is only a function of the installation geometry. This result greatly simplifies the model, because the mutual heating factor is known once the cable geometry is specified. However, when the heat generated in all cables is not equal, the mutual heating parameter is also a function of the heat generation rates as well as the geometry. Therefore, when the heat generation in the cables vary, as it does when the circulating currents in the metallic shield are unequal, the value for  $F$  should be modified appropriately.

When the effect of unequal heat generation rates are included in the mutual heating parameter, the value, as derived in [4], becomes

$$F_i = \prod_{j=1}^N \left( \frac{d_{ij'}}{d_{ij}} \right)^{\frac{q_j}{q_i}} \quad (j \neq i) \quad (2)$$

where  $F_i$  represents the mutual heating value for cable  $i$ . The symbols used in this expression are the same ones used in [3] and they are defined in the nomenclature section. Equation (2) is identical to (46) in [3] when the heat transfer rates in all cables are equal; that is when  $q_i = q_j$ . However, when the heat transfer rates differ, the value for  $F$  calculated from (2) will differ from the values proposed by Neher-McGrath.

Implementation of this new expression for the mutual heating factor obviously adds to the complexity of the thermal model. Since the value for  $F$  is not only a function of geometry, but also a function of the heat generated in each cable, its value cannot be determined until the current in each conductor and shield is known. Therefore a value for the mutual heating factor can only be determined by using an iterative scheme which assumes an approximate value for  $F$  and calculates an approximate value for the ampacity of each cable. Then the approximate ampacities are used to determine a new value for  $F$  and a corresponding new value for the cable ampacities. This iterative process continues until the ampacity values converge.

The change in the calculated ampacity value that results from the improved model for the mutual heating parameter is obviously only significant if the cables experience a reasonable difference in heat generation rates. To assess how the newly proposed mutual heating value influences the ampacity of buried cables, seven different common cable installations are considered and are summarized in Table I. These arrangements were considered because expressions for the circulating currents are known for these geometries [5].

In each of the seven cases, the metallic shield is short-circuited so that currents induced in the cables are unequal. The example selected was a 1500 kcmil, aluminum, single-conductor, 138 kV cable, with a 1/12 neutral copper shield, and operating at a loss factor of 1.0. The cables were direct buried with the shallowest one at a depth of 91.44 cm (36 inches) in a soil with a resistivity of 90 °C cm/W. The maximum operating temperature of the cables was 90 °C at an ambient temperature of 25 °C.

Figure 1 plots the percent difference in the earth thermal resistance calculated with the mutual heating factor,  $F$ , when it does not account for unequal heating and when it is evaluated by (2) as a function of the ratio of cable spacing to outer diameter of the cable,  $s/d_j$ . Negative values in Fig. 1 mean that the thermal resistance calculated by assuming equal heat generation in all of the cables is greater than the value assuming unequal generation. The differences in earth thermal resistance values are greater for small cable spacings and double, three-phase, opposed installations

TABLE I.  
INSTALLATIONS CONSIDERED FOR MUTUAL  
HEATING COMPARISON

Installation	Geometry
3 cables, right triangle	
3 horizontal cables 3 vertical cables	
2x3 and 3x2 opposed phase	
2x3 and 3x2 reversed phase	

which are conditions that magnify the difference in circulating currents.

Figure 2 shows the percent difference in calculated ampacity for the same conditions used to plot the curves in Fig. 1. These curves show that the percent differences in ampacity are all positive, or in other words, the ampacity values predicted by the computer program using the Neher-McGrath model are less than the values that result when (2) is used to calculate the mutual heating parameter in the TOAD program. Therefore, for each case considered, the Neher-McGrath model provides a conservative value for the cable ampacity and the new method for calculating the ampacity suggests that the current in all of the cables can be increased beyond the Neher-McGrath value without thermally overloading the cables.

### III. RESISTANCE OF FLUID LAYER FOR CABLES IN DUCTS AND PIPES

When cables are placed in ducts or conduits, the thermal resistance of the fluid layer that surrounds the cables must be determined in order to calculate the ampacity. Neher-McGrath presents two expressions for the resistance of this layer which are simplified forms of an earlier expression developed by Buller and Neher [6]. These expressions consider all three modes of heat transfer; conduction, convection, and radiation, between the surface of the cable and the inside surface of the duct. The total resistance of the medium can then be determined by adding the resistances in parallel.

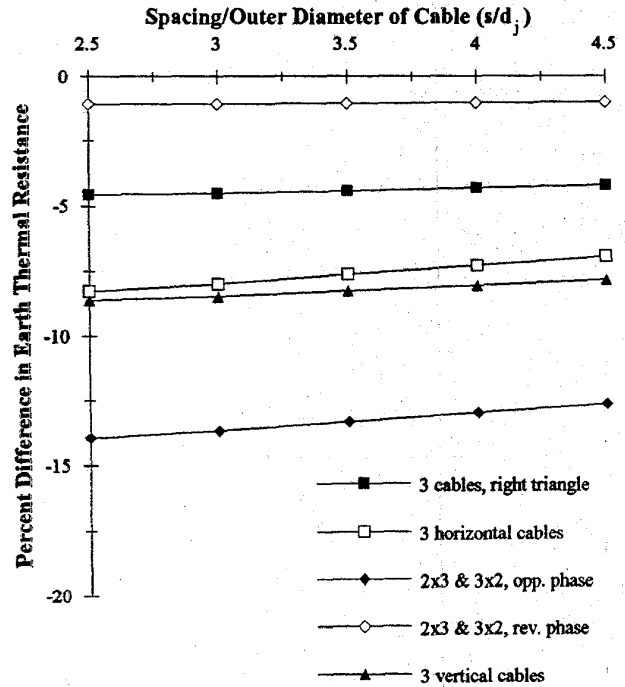


Fig. 1. Percent difference in earth thermal resistance due to unequal heat generation.

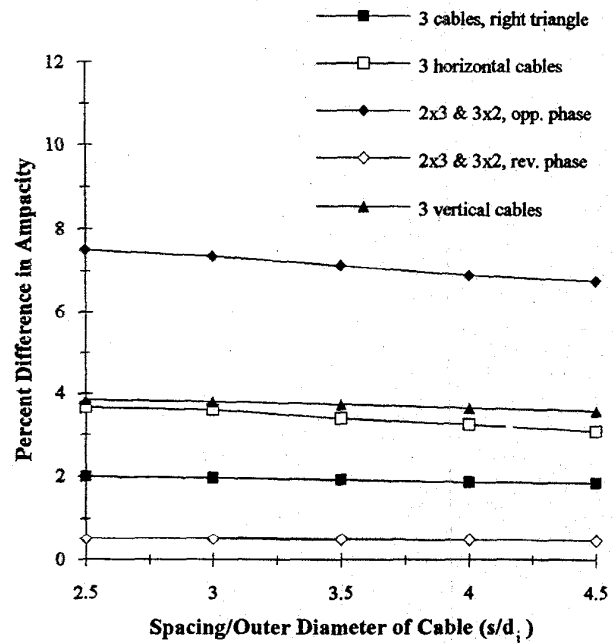


Fig. 2. Percent difference in ampacity due to unequal heat generation.

$$R_a = \left[ \frac{1}{R_{cond}} + \frac{1}{R_{conv}} + \frac{1}{R_{rad}} \right]^{-1} \quad (3)$$

The effective resistance of the air layer can be written in terms of the heat transfer,  $q$ , and the temperature difference between the outside surface of the cable and the inside surface of the duct,  $\Delta T$ .

$$R_a = \left[ \frac{q_{cond}}{\Delta T} + \frac{q_{conv}}{\Delta T} + \frac{q_{rad}}{\Delta T} \right]^{-1} \quad (4)$$

To simplify this expression and provide an expression that can be used to calculate the ampacity, several assumptions were applied by Neher-McGrath. When determining the radiative heat transfer, the emissivity of the outer surface of the jacket and the inner surface of the duct are assumed to be equal. Also, the surface area of the inside of the duct is considered to be much larger than the surface area of the cable. Buller and Neher [6] present the following expression (modified for metric units) for the thermal resistance of a gas or air layer between the cable surface and the inner wall of the duct.

$$R_a = 30.48 nm' \left[ \frac{0.0457 D_s'^{3/4} \Delta T^{1/4} P^{1/2}}{1.39 + D_s'/d_a} + \frac{0.0213}{\log_{10}(d_a/D_s')} + 0.0402 D_s' \varepsilon (1 + 0.0167 T_m) \right]^{-1} \quad (5)$$

A similar expression was also presented in [6] for oil-filled pipes.

Equation (5) is the basis of (41) in the Neher-McGrath paper. However several additional assumptions are used to express the equation in a more general form and thereby simplify the task of determining the thermal resistance of the fluid layer. First, a value for  $\Delta T$  is assumed so that iterating for a correct value for the temperature is unnecessary. A value of 20 °C is chosen for all installations except gas-filled pipe-type cables at 200 psi, where 10 °C is assumed [3]. Second, the range of  $D_s'$  is restricted to cable diameters between 1 and 4 inches when the cables are installed in ducts and between 3 and 5 inches for pipe-type cables. The complexity of the equation is further reduced by using several empirical relationships resulting in (41) in Neher-McGrath, which is (modified for metric units),

$$R_a = \frac{30.48 nm' A}{1 + (B + CT_m)(D_s'/2.54)} \quad (6)$$

where  $T_m$  is the mean air temperature in the duct.

The constants  $A$ ,  $B$ , and  $C$  are presented in Table VII in [3] for a variety of installation conditions. Neher and McGrath further assume a mean temperature,  $T_m$ , of the intervening medium of 60 °C which reduces (6) to the following form.

$$R_a = \frac{30.48 nm' A'}{(D_s'/2.54) + B'} \quad (7)$$

The constants  $A'$  and  $B'$  are also listed in Table VII [3]. According to this expression, the thermal resistance between the cable surface and the inner wall of the duct is only a function of the outer diameter of the cable, the number of conductors within the duct, and the thermal properties of the duct which are incorporated into the values for the constants  $A'$  and  $B'$ . The assumptions used in the derivation of (7) greatly simplify its form. However they also reduce its accuracy and even can produce erroneous results. For example, (7) suggests that the resistance of the liquid layer is independent of the thickness of the fluid gap.

To improve the accuracy of the expression for the thermal resistance, the Buller and Neher expression is revised by eliminating the restrictive assumptions and using more up-to-date convective correlations. The three modes of heat transfer are considered individually and (4) is used to determine an expression for the total thermal resistance between the cable surface and the inner wall of the duct. The cable installation is modeled as concentric cylinders. Also, the outer surface of the cable and the inner surface of the duct are assumed to be isothermal. These assumptions are not precisely met in practical installations, especially for non-metallic ducts that have a very high thermal resistivity and can be hot where they touch the cable and cool in other regions. Metallic pipes have a lower thermal resistivity and therefore the temperature of the pipe is more uniform. The installations are also assumed to be infinitely long and therefore the heat from the cable is transferred only in the radial direction.

The first mode of heat transfer considered is conduction across the layer. The expression for thermal resistance due to conduction between concentric cylinders is used in an unmodified form but it is converted so that it is expressed in terms of metric units.

$$\frac{q_{cond}}{\Delta T} = \left[ 0.366 nm' \rho_a \log_{10} \left( \frac{d_a}{D_s'} \right) \right]^{-1} \quad (8)$$

Next considering the radiant heat transfer that occurs across the gap, the Stefan-Boltzmann equation [7] is used to determine the heat transfer between concentric cylinders,

$$\frac{q_{rad}}{\Delta T} = \frac{\sigma \left[ (T_j + 273)^4 - (T_a + 273)^4 \right]}{\Delta T \left( \frac{\rho_j}{\varepsilon_j A_j} + \frac{1}{A_j F_{j-a}} + \frac{\rho_a}{\varepsilon_a A_a} \right)} \quad (9)$$

where  $\rho$  represents the reflectivity of the surfaces. Simplifying the expression by assuming gray, diffuse surfaces and also considering the possibility of multiple conductors within the enclosed duct results in the following expression.

$$\frac{q_{rad}}{\Delta T} = \frac{1.781 \times 10^{-11} D_s' \left[ (T_j + 273)^4 - (T_a + 273)^4 \right]}{\Delta T n n' \left( \frac{1 - \varepsilon_j}{\varepsilon_j} + \frac{(1 - \varepsilon_a) \left( \frac{D_s'}{d_a} \right) + 1}{\varepsilon_a} \right)} \quad (10)$$

This equation is applicable for installations which involve air or gas layers. For oil-filled pipes, the effect of radiation is negligible.

The third mode of heat transfer is convection in the fluid layer. The thermal resistance of the air layer can be accurately calculated by using a more recent correlation for the convective heat transfer coefficient between concentric cylinders. Raithby and Hollands [8] have proposed a correlation that is valid for concentric, isothermal, long, horizontal cylinders in which the diameter of the inner cylinder is large compared to the width of the air gap. Written in terms of dimensionless groups, this expression is

$$\frac{q_{conv}}{\Delta T} = \frac{2 \pi k_{eff}}{\ln(d_a/D_s')} \quad (11)$$

where

$$k_{eff} = 0.386 k \left[ \frac{Pr}{0.861 + Pr} \right]^{1/4} (Ra_c^*)^{1/4}$$

$$(Ra_c^*)^{1/4} = \frac{\ln(d_a/D_s')}{\delta^{3/4} \left[ (D_s')^{-3/5} + (d_a)^{-3/5} \right]^{5/4}} (Ra_\delta)^{1/4}$$

$$Ra_\delta = \frac{g \beta (T_j - T_a) \delta^3 Pr}{\nu^2}$$

$$\delta = \frac{1}{2} (d_a - D_s')$$

Substituting these expressions into (11) for the air layer and simplifying results in the following expression.

$$\frac{q_{conv}}{\Delta T} = \frac{2.4253 \left[ \frac{Pr}{0.861 + Pr} \right]^{1/4}}{n n' \rho_a} \left[ (D_s')^{-3/5} + (d_a)^{-3/5} \right]^{-5/4} \left[ \frac{g \beta (T_j - T_a) Pr}{\nu^2} \right]^{1/4} \quad (12)$$

The thermal properties that occur in (12) are assumed to be constant over the small range of expected operating temperatures. Appropriate properties for air, gas, and oil are listed in Table II.

To illustrate the difference in thermal resistance of the layer between the cable and duct that results from using Neher-McGrath's equation (41A) and the value predicted by the new expression, a typical cable installation is considered and the thermal resistance values are plotted in Fig. 3. This figure assumes a 250 kcmil, copper, single-conductor, 35 kV cable with a 12, #14 AWG copper wire shield buried in a 0.635 cm (0.25 inch) thick fiber duct, at a depth of 91.44 cm (36 inches), a soil thermal resistivity of 60 °C cm/W, and operating with a loss factor of 1.0. The maximum operating temperature of the cables was 90 °C, while the ambient temperature was 25 °C. For this installation, both the duct and the cable surface are assumed to have an emissivity of 0.95. The thermal resistance values are plotted as a function of the ratio of outer diameter of the cable to inner diameter of the duct,  $d_j/d_a$ . The Neher-McGrath equation predicts a thermal resistance that is independent of the thickness of the air layer that surrounds the cable, while the new expression predicts the more logical result that the resistance increases as the thickness of the air layer increases to a maximum resistivity at a ratio of  $d_j/d_a$  of about 0.35, for the installation considered. As the thickness of the air layer increases further, motion of the air surrounding the cable increases and these convective currents produce a decrease in the thermal resistance in the air layer as indicated in Fig. 3.

Table II.  
THERMAL PROPERTIES FOR THE INTERVENING MEDIA [6,7]

Quantity	Air (1 atm)	Gas (200 psi)	Oil
$\rho$ - thermal resistivity (°C cm/W)	3550	3900	715
$\beta$ - thermal coeff. of expansion (1/°C)	0.00308	0.00310	0.00068
$Pr$ - Prandtl number (dimensionless)	0.715	0.7567	1126.1
$\nu$ - kinematic viscosity (cm <sup>2</sup> /s)	0.1880	0.01303	0.8278

In addition to the dependence on the thickness of the air layer, the thermal resistance is shown to increase as the ratio of  $\epsilon_j / \epsilon_d$  decreases, while the Neher-McGrath result is independent of the emissivity ratio. Fig. 4. shows the effect of surface emissivity on the thermal resistance. In many installations, it is not reasonable to assume that the emissivities of both surfaces are equal. The emissivity ratio,  $\epsilon_j / \epsilon_d$ , equal to 1.0 is typical of properties of a cable with a polyethylene jacket in a fiber duct, while a ratio of 2.0 is a reasonable ratio for cables in a metallic pipe. Curves are shown in Fig. 4 for a ratio of jacket to duct surface emissivity of 1 and 2. As shown in the figure, removal of the restriction of equal emissivities can produce thermal resistivities that are higher than those previously calculated, thereby producing a positive percent difference in the thermal resistance.

When these improved values for thermal resistances are used in the TOAD ampacity program, significantly different ampacity values can result. The ampacity values corresponding to the conditions used in Fig. 3 are shown in Fig. 4. A program using the Neher-McGrath model provides conservative ampacity values over the entire range

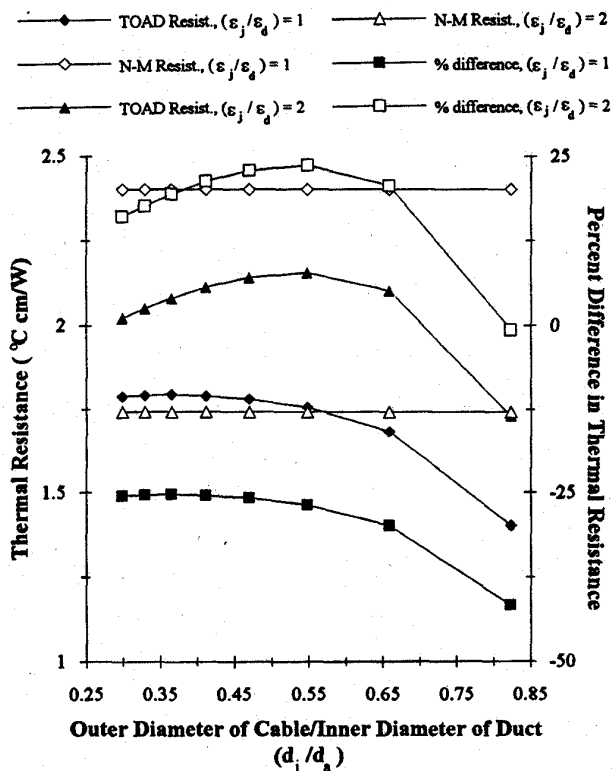


Fig. 3. Thermal Resistance calculated by Neher-McGrath expression and new expression for thermal resistance.

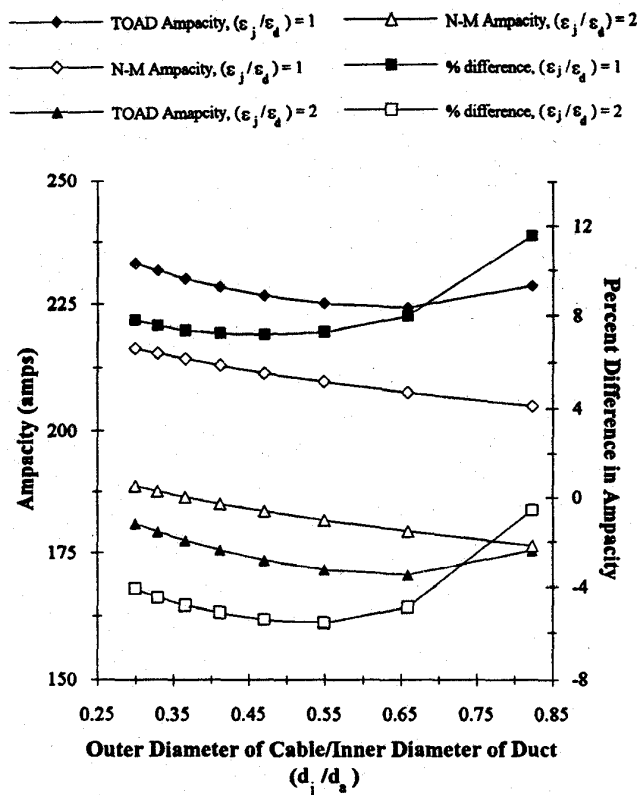


Fig. 4. Ampacity of installation calculated by Neher-McGrath model and model using new expression for thermal resistance of fluid layer.

of cable and duct diameters, for  $\epsilon_j / \epsilon_d$  equal to 1.0. For small air gaps, the percent difference in ampacity between the Neher-McGrath model and the new model can be as large as 11 percent. Therefore, for this particular cable and installation geometry, the ampacity of the circuit as predicted by the new model is over 11 percent greater than the value predicted by the Neher-McGrath model when the cable diameter is approximately 85 percent of the duct diameter.

For the same installation, but with emissivity ratios,  $\epsilon_j / \epsilon_d$ , that are not 1.0, the ampacity predicted by Neher-McGrath is no longer conservative, as also shown in Fig. 4. For an emissivity ratio of 2.0, the ampacity based on the Neher-McGrath model is higher than the current-carrying capacity of the installation as predicted by the new model for cable diameters that are less than approximately 0.85 times the diameter of the duct. Neglecting the effects of unequal surface emissivities could result in higher cable temperatures, therefore shortening the life of the cable.

#### IV. RESISTANCE OF RECTANGULAR DUCT BANKS

The final modification to the ampacity model involves the calculation of the thermal resistance of a rectangular region that surrounds the cable. This material can be assumed to be a concrete duct bank or backfill material used to reduce the thermal resistance of the region adjacent to the cables. For a rectangular region with a shorter dimension of  $x_b$ , a longer dimension of  $y_b$ , and a thermal resistivity of  $\rho_{db}$ , Neher-McGrath suggest the following expression ((44A) in [3]) to calculate the effective thermal resistance of the rectangular region and soil (modified for metric units).

$$R_e = 0.366 \rho_{db} m' \left[ \log_{10} \left( \frac{D_x}{d_e} \right) + (LF) \log_{10} \left( \left( \frac{4L}{D_x} \right) F \right) \right] + 0.366 (\rho_e - \rho_{db}) m' N (LF) G_b \quad (13)$$

This expression was derived by replacing the actual rectangular region with a circular region which possesses an equivalent thermal resistance. This assumption allows the two-dimensional heat transfer problem to be reduced to an approximate one-dimensional analysis. The radius of the equivalent duct bank can be expressed as

$$\log_{10} r_b = \frac{1}{2} \frac{x_b}{y_b} \left( \frac{4}{\pi} - \frac{x_b}{y_b} \right) \log_{10} \left( 1 + \frac{y_b^2}{x_b^2} \right) + \log_{10} \frac{x_b}{2} \quad (14)$$

The geometric factor,  $G_b$ , in (13) is determined from using the Kennelly formula [3].

$$G_b = \log_{10} \left[ \frac{L_b}{r_b} + \sqrt{\left( \frac{L_b}{r_b} \right)^2 - 1} \right] \quad (15)$$

The surface of the duct bank is assumed to be transformed into an isotherm of radius  $r_b$  and the heat flow through the duct bank is assumed to be independent of angular direction. These assumptions are not particularly accurate, because the lower surfaces of the duct bank are hotter than the upper surfaces. Also, more heat flows toward the surface of the earth, because the thermal resistance of the earth layer over the cables is significantly less than the resistance of the earth under the cables.

The errors introduced by transforming a rectangular region into one which has a circular cross-section are less when the duct bank or thermal backfill layer are nearly square ( $x_b = y_b$ ). However when the region is very slender, the results predicted by (13)-(15) are no longer accurate. In order to improve on the accuracy of these equations for width-to-height ratios not close to 1.0, a conformal mapping

technique was used [9] to predict the value for the thermal resistance of the rectangular region.

The conformal mapping technique provides a conduction shape factor,  $S$ , and an effective resistivity value,  $\rho_{eff}$ , that differs for each cable in the installation. The effective thermal resistance,  $R_{w/o-LF}$ , not accounting for the loss factor can be expressed as

$$R_{w/o-LF} = \left( \frac{\rho_{eff}}{S} \right) \quad (16)$$

Once the value for  $R_{w/o-LF}$  is determined, it can be written in a form similar to (13), where the loss factor is 1.0. This expression can then be combined with the Neher-McGrath equation, (13), to arrive at the following expression for the effective earth resistance,  $R_e$ , that is valid for any loss factor.

$$R_e = (R_{w/o-LF}) LF + 0.366 \rho_{db} m' (1 - LF) \log_{10} \left( \frac{D_x}{d_e} \right) \quad (17)$$

By comparing the thermal resistances provided by (17) and the corresponding equations suggested by Neher-McGrath, (13)-(15), it is possible to assess the errors in the old method of calculating the ampacity when cables are routed through rectangular duct banks. The thermal resistances shown in Fig. 5 illustrate the differences in the two models for the case of three, horizontally spaced 1500 kcmil, single-conductor, 138 kV, aluminum cables with an open circuited, copper shield that are buried in a duct bank with a resistivity of 80 °C cm/W, and operating at a loss factor of 1.0. The cables are placed in 15.24 cm (6.0 in) ducts at a depth of 91.44 cm (36 inches) below the surface of the earth and separated by a distance of 30.48 cm (12.0 inches). The operating temperature of the cables was 90 °C and the ambient earth temperature was assumed to be 25 °C. The curves provide results for a range of soil thermal resistivities and different values for  $x/y$ .

The results in Fig. 5 show that the percentage difference in the effective thermal resistivity of the soil/duct bank layer is small as long as the duct bank is nearly square in cross-section. However, when the duct bank is slender, ( $x/y$ ) < 0.5, the difference between the two methods can become significant. For example as  $x/y$  approaches 0.2, the Neher-McGrath method of calculating the thermal resistance gives a value that is as much as 8 percent less than the conformal mapping results. The percent error at this  $x/y$  ratio increases as the difference between the duct bank and native soil thermal resistance increases. This same general trend was also shown using a finite element technique [10]. The corresponding percent difference in ampacity between the Neher-McGrath program and the TOAD program is shown in Fig. 6.

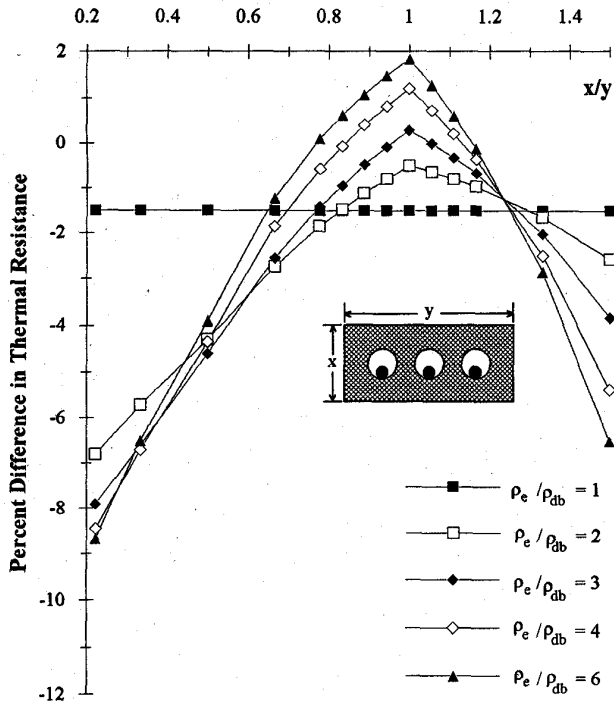


Fig. 5. Percent difference in thermal resistance for various duct bank heights with a width of 91.44 cm (36 inches).

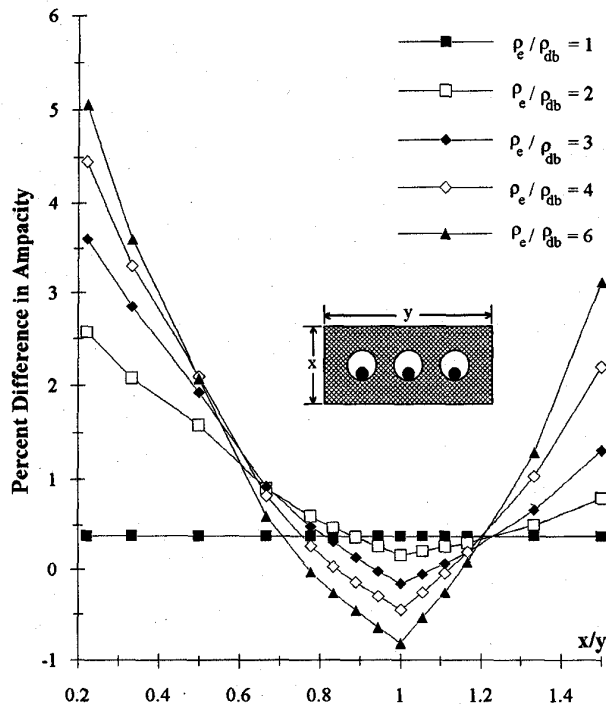


Fig. 6. Percent difference in ampacity for various duct bank heights with a width of 91.44 cm (36 inches).

## V. CONCLUSIONS

Three refinements to the Neher-McGrath ampacity model are proposed in this paper. The first involves the mutual heating parameter and it removes the assumption that all cables must generate the same amount of heat. This improvement therefore increases the accuracy of ampacity calculations when the circulating currents in the metallic shield produce unequal heating among the cables. For several typical cable installations, this refinement to the Neher-McGrath model suggests that the ampacity value can be increased by up to 7 percent.

The second refinement involves the correlation used to calculate the thermal resistance of the air or oil layer that surrounds pipe-type cables or cables in ducts. By replacing old heat transfer correlations by more up-to-date ones and by expanding the model to cases not considered by Neher-McGrath, more accurate and more realistic ampacity values can be calculated. Applying the new model to typical installations has shown that the ampacity can be increased up to 11 percent in some cases and must be reduced by up to 5 percent in others. The case for which the new model recommends a reduction in the ampacity is of particular importance, because it suggests that the Neher-McGrath model does not provide a conservative ampacity, but one which could lead to an unexpected high cable temperature.

The final refinement considers the calculation of the thermal resistance of a rectangular duct bank. A proposed expression for the thermal resistance is based on a conduction shape factor that can be calculated from a conformal mapping technique. Unlike the Neher-McGrath treatment of the type of installation, the new expression is not limited to duct banks that have a nearly square cross-section. For typical cables installed in rectangular duct banks, the revised expression for thermal resistance predicts ampacities that are up to about 5 percent greater than, or about 1 percent less than, the Neher-McGrath values.

## VI. ACKNOWLEDGMENTS

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## VII. NOMENCLATURE

$A$  - surface area ( $\text{cm}^2$ )

$A, B, C, A', B'$  - constants for calculation of thermal resistance of air layer

$D'_s$  - effective diameter of multiple cables in contact (cm)

$D_x$  - diameter at which loss factor becomes significant (cm)

$d$  - outer diameter (cm)

$d_{ij}$  - distance from center of cable i to center of cable j (cm)  
 $d_{ij}'$  - distance from center of cable i to center of mirror image of cable j (cm)  
 $F$  - mutual heating factor (dimensionless)  
 $F_{i,j}$  - shape factor from surface i to surface j (dimensionless)  
 $G_b$  - geometric factor (dimensionless)  
 $g$  - gravitational constant (980 cm/s<sup>2</sup>)  
 $k$  - thermal conductivity of the earth (W/°C cm)  
 $LF$  - loss factor (dimensionless)  
 $L$  - depth of shallowest cable below surface of the earth (cm)  
 $N$  - total number of cables in arrangement  
 $n$  - number of cables per duct  
 $n'$  - number of conductors per cable  
 $P$  - pressure (atm)  
 $Pr$  - Prandtl number (dimensionless)  
 $q_i$  - heat transfer per unit length for cable i (W/cm)  
 $R$  - thermal resistance (°C cm/W)  
 $Ra_c^*$  - modified Rayleigh number for concentric cylinders (dimensionless)  
 $Ra_\delta$  - Rayleigh number based on thickness of fluid layer (dimensionless)  
 $r_b$  - effective radius of duct bank (cm)  
 $S$  - shape factor for a particular cable (dimensionless)  
 $s$  - spacing between adjacent cables (cm)  
 $T$  - temperature (°C)  
 $x$  - height of duct bank (cm)  
 $x_b$  - length of short side of duct bank (cm)  
 $y$  - width of duct bank (cm)  
 $y_b$  - length of long side of duct bank (cm)

#### Greek Symbols

$\beta$  - thermal coefficient of expansion (1/°C)  
 $\delta$  - thickness of fluid layer (cm)  
 $\epsilon$  - thermal emissivity (dimensionless)  
 $\nu$  - kinematic viscosity (cm<sup>2</sup>/s)  
 $\rho$  - thermal resistivity (°C cm/W)  
 $\sigma$  - Stefan-Boltzmann constant (5.67 x 10<sup>-8</sup> W/m<sup>2</sup> K<sup>4</sup>)

#### Subscripts

$a$  - air  
 $cond$  - conduction  
 $conv$  - convection  
 $d$  - duct or pipe  
 $db$  - duct bank or backfill  
 $e$  - earth  
 $eff$  - effective  
 $j$  - jacket  
 $m$  - mean  
 $rad$  - radiation  
 $w/o-LF$  - without loss factor

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## IX. BIOGRAPHY

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W. Z. Black received his B.S. and M.S. in Mechanical Engineering from the University of Illinois and his Ph.D. in M.E. from Purdue University. He is currently Regents Professor and Georgia Power Distinguished Professor of Mechanical Engineering at Georgia Institute of Technology in Atlanta. His research effort is concentrated in the area of heat transfer from electrical and electronic equipment. He has been Principal Investigator for several EPRI ampacity projects and he is active in IEEE committee work relating to the thermal ratings of overhead and underground conductors.

## Discussion

L.H.Fink (ECC, Inc., Fairfax, Virginia). The discussor had the privilege of sharing an office and working closely with J.H.Neher during the decade of the 1950s when the Neher-McGrath paper was being written. Neher was painstaking, conscientious, and rigorous in his work, never satisfied with an interim or approximate solution. Accordingly, he never considered "Neher-McGrath" to be the last word, and would have been delighted to know that further work and refinements are still being pursued.

Having said this, this discussor has serious reservations about the conclusions reached by the authors of the present paper. Neher recognized the needs, not only to ground his work on the best available scientific understanding of the phenomena with which he was dealing, and not only also to make simplifying assumptions necessary to provide practicable solution techniques, but also, and importantly, to validate the latter by adjusting them to fit laboratory and field test data. This may be seen in his earlier papers on which the Neher-McGrath paper was based, e.g. the authors' reference 6. The underlying theory and accompanying linearizations provide confidence that the relationships may be used for interpolation within a range of validity that has been established. Improved theoretical relationships may provide a basis of increasing the range within which interpolation is valid, but such extensions should not be adopted without subjecting them to validation by test data. The Neher-McGrath results will remain valid within their range of validity just because they were so validated. The authors' assertion in Section III that some of the Neher-McGrath assumptions "reduce accuracy and even can produce erroneous results" can be so *only* if the Neher-McGrath equations are used outside the limits within which they were validated.

Just one reservation will be mentioned. In Section III of the paper, the authors (as did Neher-McGrath) ground their analysis in concentric cylinder geometry. It is evident, however, that this is rarely if ever true in practice: individual cables in ducts will be resting on the bottom of the interior duct wall; individual phase cables in pipe will either be (randomly) cradled or triangular at low loads, or birdcaged at high loads. Whether we are dealing with convection and radiation (for cables in duct), or with convection and conduction (for high-pressure oil pipe-type cable, calculations based on concentric configuration must be treated with the greatest caution. In either case, the effective distance between the cable surface and the duct or pipe wall will not vary directly with the respective diameters. *Caveat*: It cannot be unfair to echo the authors' own words: such assumptions, *if the results are not validated against test data*, can "reduce accuracy and even can produce erroneous results."

In short, until the results of this paper have been validated and, if necessary, corrected by reference to appropriate laboratory or field test data, they should be considered only as hypotheses.

In Section IV, the authors undertake to correct the Neher-McGrath factor  $G_b$  for duct banks. Neher was well aware of the limited validity of that factor as given in the paper. In 1958, Fink and Smerke [A] published an analysis of the effect of (backfilled) trench dimensions on the effective earth thermal resistance around pipe type cables, using conformal mapping. In an appendix, this analysis was extended (at Neher's request) to duct configurations, and was later used internally to correct  $G_b$  for various duct bank configurations. The authors should provide the conformal mapping relationships that were used in obtaining their results, in order to enable independent assessment.

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Manuscript received February 8, 1995.

George J. Anders. Ontario Hydro. Most utilities in the USA used the Neher-McGrath (NM) paper as a basis for computation of ampacities of electric power cables. The paper has been published in 1957 and is based on the analytical and experimental work performed in the preceding 30 years. As the authors of the discussed paper rightly pointed out, the NM paper is based on number of simplifications and uses several correlations which are now dated. The authors of the discussed paper are to be congratulated on attempting to update some of the computational procedures used in North American standards which are based on the NM paper.

The purpose of this discussion is to review the contents of the paper in the context of published and standardized information and to supply additional information on the topics addressed in the paper. The following is a discussion of the three contributions introduced by the authors.

## Mutual Heating Factor

The computation of the external thermal resistance discussed under the heading of Mutual Heating Factor is one of the topics dealt with in an international standard. The analysis of unequally loaded cables leading to Equation (2) in the paper is described in the IEC Standard 287 (1982) and the approach proposed there is identical to the one proposed to the authors. To see that, we first observe that for several loaded cables placed underground, we must deal with superimposed heat fields. The principle of superposition is applicable if we assume that each cable acts as a line source and does not distort the heat field due to the other cables. Therefore, in the NM and the discussed paper, it is assumed that the cables are spaced sufficiently apart so that this assumption is approximately valid. The axial separation of the cables should be at least two cable diameters. The case when the superposition principle is not applicable is also dealt with in IEC Publication 287 (1982)..

The method suggested in the IEC Publication 287 for the calculation of ratings of a group of cables set apart is to calculate the temperature rise at the surface of the cable under consideration by the other cables of the group, and to subtract this rise from the value of  $\Delta T$  used in the equation for the rated current. An estimate of the power dissipated per unit length of each cable must be made beforehand, and this can be subsequently amended as a result of the calculation where this becomes necessary, which also applies to equation (2) of the discussed paper.

Section 9.3.1 of IEC 287 gives the following equation for the temperature rise at the surface of the cable  $p$  produced by the power  $q_k$  watt per unit length dissipated in cable  $k$  (a unity load factor is considered below, but the reasoning

applies also in the case of the nonunity load factor):

$$\Delta T_{kp} = \frac{\rho_s}{2\pi r} q_k \ln \frac{d'_{pk}}{d_{pk}} \quad (D1)$$

The distances of  $d_{pk}$  and  $d'_{pk}$  are measured from the centre of the  $p^{\text{th}}$  cable to the centre of cable  $k$ , and the centre of the reflection of cable  $k$  in the ground-air surface, respectively.  $\rho_s$  is the thermal resistivity of the soil. Equation D1 is easily developed based on the Kennelly's theorem. All the variables are expressed in metric units.

Thus, the temperature rise  $\Delta T_p$  above ambient at the surface of the  $p^{\text{th}}$  cable, whose rating is being determined, caused by the power dissipated by the other (q-1) cables in the group, is given by:

$$\Delta T_p = \Delta T_{1p} + \Delta T_{2p} + \dots + \Delta T_{kp} + \dots + \Delta T_{qp} \quad (D2)$$

with the term  $\Delta T_{pp}$  excluded from the summation.

The value of  $\Delta T$  in equation 5 in the NM paper for the rated current is then reduced by the amount of  $\Delta T_p$  and the rating of  $p^{\text{th}}$  cable is determined using a value of  $R_p$  corresponding to an isolated cable at position  $p$ . This calculation is performed for all cables in the group and is repeated where necessary to avoid possibility of overheating any cable.

Substituting equation D1 into the right-hand-side of equation D2 and taking the definition of the external thermal resistance of a cable (the ratio of the cable surface temperature rise to the total cable losses), the following general expression for the external thermal resistance of cable  $p$  is obtained:

$$R_p^e = -\frac{\rho_s}{2\pi r} \left( \ln(u + \sqrt{u^2 - 1}) + \frac{1}{q_p} \sum_{\substack{k=1 \\ k \neq p}}^q q_k \ln \frac{d'_{pk}}{d_{pk}} \right) \quad (D3)$$

where  $u = 4L/d_e$ .

It is evident that the second term in the summation in equation D3 is equivalent to equation 2 in the paper. Moreover, it is more practical in computer application to use directly equation D2 as described above than to use equation D3.

### Resistance of Fluid Layer for Cables in Ducts and Pipes

The second refinement introduced in the paper deals with the calculation of the resistance of fluid layer for cables in ducts and pipes. In the opinion of this discussor, the updating of the computation of the thermal resistance of air for cables in ducts is an important contribution of the paper. However, one should bear in mind that the method for the computation of the thermal resistance of the fluid in a duct or pipe introduced by Neher and McGrath and adopted in the IEC Standard 287 is very useful precisely because it is simple and does not depend on the unknown cable surface temperature. If the method proposed by the authors is adopted, then there is no need to compute the thermal resistance of the fluid, but it is sufficient to solve the energy balance equation 4 in the paper which can be stated as (the notation is the same as in the NM paper):

$$I^2 R_{dc} (1 + Y_c + Y_s + Y_p) = q_{conv} + q_{cond} + q_{rad} \quad (D4)$$

The conductive, radiative and convective heat transfer rates are obtained from equations 8, 10 and 12 in the paper, respectively. Equation D4 has three unknown parameters: the conductor current, temperature of the cable jacket and temperature of the fluid in the duct. The second equation relates the current in the conductor with the jacket temperature:

$$T_c - T_j = n(I^2 R_{dc} (1 + Y_c + Y_s + Y_p) \cdot R_i + \Delta T_d) \quad (D5)$$

where  $\Delta T_d$  is the cable surface temperature rise due to dielectric losses and  $R_i$  is the equivalent internal thermal resistance of the cable.

As for the third unknown variable, there are two ways to approach the problem. One, is to assume that  $T_a$  is equal to the mean value of jacket and ambient

temperatures, and the second is to write full set of energy balance equations. The last approach is more accurate and is formulated for a general case of cables enclosed in walls in the paper presented at the same Winter Meeting as the discussed paper (Anders, 1995). Since the authors of the paper face the same problem of defining the value of  $T_a$ , it will be appreciated if they could reveal how they define this temperature.

The solution of equations D4 and D5 yields the conductor rated current  $I$  and the jacket temperature  $T_j$  (assuming that  $T_a$  is defined in terms of  $T_j$ ).

With regards to the value of  $T_a$ , one more point may be of interest. The temperature  $T_a$  appearing in equations 9 and 10 should actually be the temperature of the inner wall of the duct or pipe while the value in equation 12 is actually the temperature of the air. Could the authors comment on this?

Continuing with the problem of cables in ducts or pipes, equation 9 in the paper originates from the following standard definition of the radiative heat transfer:

$$q_{rad} = A_j F_{j,d} \sigma (T_j^{*4} - T_d^{*4}) \quad (D6)$$

where:

- $\sigma$  = Boltzmann constant, equal to  $5.667 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4)$ ;
- $T_d^*$  = the average temperature of the wall inner surface, K, (asterisk denotes absolute temperatures)
- $F_{j,d}$  = thermal radiation shape factor, its value depends on the geometry of the system,
- $A_j$  = is the area of the cable surface effective for heat radiation, ( $\text{m}^2$ ); for unit length of the cable.

Two points are worth mentioning in the context of the discussed paper: (1) the value of the radiation shape factor, and (2) the value of the variable  $A_j$ .

The radiation shape factor is obtained considering two long concentric cylinders. In the case of a single cable in the riser, we have:

$$F_{j,dw} = (1 + \sigma_j / \epsilon_j + A_j \sigma_d / A_d \epsilon_d)^{-1} \quad (D7)$$

where:

- $A$  = area per unit length
- $\sigma_j$  = the reflectivity of the cable outside surface;
- $\epsilon_j$  = the emissivity of the cable outside surface;
- $\sigma_d$  = the reflectivity of the wall inner surface;
- $\epsilon_d$  = the emissivity of the wall inner surface;

It is evident that equation D7 is equivalent to the formulation in equation 10 of the paper where a group of cables is replaced by an effective diameter  $D_j$  of a single cable and gray, diffuse surfaces are assumed. However, it is insufficient to just use an effective diameter in equation D7 when several cables are in installed in one duct. For the case of several cables in a duct, equation D7 takes the form:

$$F_{j,d} = (1 + \sigma_j / \epsilon_j + \zeta A_j \sigma_d / A_d \epsilon_d)^{-1} \quad (D8)$$

For the case of three cables in a duct, we have:

$$\zeta = \frac{3}{1 - \frac{6\pi A_j \epsilon_j \sigma_d}{A_d \epsilon_d}} \quad (D9)$$

The second point concerns computation of  $A_j$ . When several cables are located in one duct, the mutual radian area between them must be subtracted from the area radiating to the duct inner surface. The most common installations have either one or three cables inside the guard. For example, for three cables in touching trefoil formation, we have (Weedy, 1988):

$$A_j = 3\pi d_e - 3 \cdot 0.618 d_e \quad (D10)$$

## Resistance of Rectangular Duct Banks

The third topic introduced in the paper deals with computation of the external thermal resistance of cables in duct banks or backfills. The NM method assumes that the duct bank surface is isothermal and as pointed out in the NM paper and in IEC 287, it is valid for the ratio of duct bank dimensions in the range of 1/3 to 3. In the studies performed by the authors, the results of the method presented in the paper are compared with the NM results for this ratio equal to 0.2.

The approach proposed by the authors is not well explained and the crucial information how to obtain the conduction shape factor  $S$  and the effective resistivity value  $\rho_{eff}$  is withheld. In this context, the following comments are in order. There are several drawbacks to the conformal transformation method used by the authors (see reference CIGRE, 1985 for the full description of this technique). The major one is that the equations describing the transformed network are equivalent to finite difference equations obtained by discretizing the heat equation in the transformed plane and, hence, the complexity of a numerical solution of the heat conduction problem is not avoided. Another drawback is that both the earth and cable surfaces are assumed to be isothermal. In addition, transformation point by point of the boundaries between regions with different resistivities is very laborious and the resulting computer software cannot handle efficiently more than four cables in one installation. All these limitations can easily be overcome by the application of the finite element method. The nonuniform soil conditions and nonisothermal boundaries are handled naturally by this method. The computational efficiency of this approach is also quite satisfying. With the presently available personal computers, calculations involving networks with several thousand nodes can be performed in a matter of minutes.<sup>1</sup> The finite element method is described in (Anders and Coates, 1995).

The limitation of the NM method for the calculation of the external thermal resistance of cables in duct banks or backfills has been recognized by several authors. El-Kady and Horrocks (1985) used the finite element method to develop the values of the geometric factor for thermal envelopes with ratios beyond the range specified above. Their approach has an advantage of providing a table of geometric factors for a very wide range of cable installations. This table is used directly in the cable ampacity program written by the discussor (Anders et al., 1991) and the results are very satisfactory. In later works of El-Kady and others (El-Kady et al., 1988, Tarasiewicz et al., 1987) the basic method of Neher and McGrath was extended to remove the assumption that the external perimeter of the rectangle is isothermal.

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<sup>1</sup> The discussor performed computations on the network of 7000 nodes in less than 5 minutes using 66 MHz, 486 PC.

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**Vincent Morgan** (CSIRO Division of Applied Physics, Sydney, Australia): It is appropriate that the often-used heat transfer correlations for cables, given by Neher and McGrath [3] nearly forty years ago, should be re-assessed in the light of recent research. However, the transition from a set of mixed units to metric units runs the risk of generating errors. One such error appears to have occurred in (5), which is converted from (19) in reference [6]. The coefficient in the numerator of the first term within the square brackets on the right-hand side should read  $0.092 / (2.54)^3 = 0.0457$ , instead of  $0.092 / (2.54) = 0.0362$ .

The correlation (11) given by Raithby and Hollands [8] for the natural convective heat transfer from isothermal concentric horizontal cylinders agrees closely with the numerical results obtained by Farouk and Guceri [A]. The range of the Rayleigh number in (11) is  $10^2 - 10^7$ . The type of flow in the annulus depends on the ratio of the inner diameter to the thickness of the gap and the Grashof number [B].

It is stated that Fig. 3 (not Fig. 4) shows that the calculated thermal resistance to the annular air layer increases as the ratio  $\epsilon_1 / \epsilon_2$  decreases. The labeling of the curves indicates that this is true for the Neher-McGrath results, but not for the authors' results.

The authors have noted that the surface of the duct bank is not isothermal, but they have assumed the surface to be transformed into a circular isotherm. Other researchers [C] have assumed the more realistic condition of uniform flux, and have used a numerical method to derive modified geometric factors for various configurations of the duct bank.

Neher and McGrath [3] mentioned the effect of solar heating at the surface of the soil, but they did not elaborate on this, instead preferring to take the ambient temperature at the depth of the cable. Would the authors care to comment?

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Manuscript received March 2, 1995.

**A. Ernst and D. W. Purnhagen** (Underground Systems, Inc., Armonk, NY): Our comments on this paper can be summarized as follows:

1) Equation 2 for the mutual heating factor  $F$  in this paper is mathematically correct and calculations using this equation *do agree* with calculations based on the Neher and McGrath paper for cables with unequal losses. The authors have mis-applied equation 44 of the Neher and McGrath paper which leads them to the incorrect conclusion that a discrepancy exists between their equation and the Neher and McGrath method.

2) The Neher and McGrath equations for thermal resistance from cable surface to duct or pipe wall were derived by fitting equations to data. Any alternative equation should fit the same data but the authors offer no experimental substantiation for their formulation. We have reviewed available data and reach the following conclusions:

The equations in this paper do not agree with available

data for cables installed in conduits where air is the intervening medium or for high pressure gas filled pipe type cables.

Use of these equations for cable in conduit or high pressure gas installations can lead to substantial errors in cable ampacity or temperature computations.

However the equations do agree with the data for pipe type cables where dielectric liquid is the intervening medium.

In the first part of this paper the authors state that the Neher and McGrath paper is incorrect for cases where the cables have unequal losses such as occurs when single conductor cables are installed in separate ducts and circulating currents flow in the metallic shields or sheaths. This statement is not correct.

Equation 44 of the Neher and McGrath paper (equation 1 of the Black and Sellers paper) applies only for the case of a single cable or a group of cables with equal losses. Apparently the authors have used this equation to compute what they refer to as the "Neher-McGrath" computations in making the comparisons of Figures 1 and 2 of their paper.

For the case of unequal losses equations 1A, 9A, 48, 49 and 50 of the Neher and McGrath paper apply. Each cable or group of cables with equal losses is considered separately. The mutual heating due to the remaining cables is added to the earth temperature at the location of the cable being rated (or subtracted from the allowable temperature rise to compute ampacity). When cables have different losses an iterative or simultaneous solution with an equation for each cable may be required. However if the ratio of losses is known a single equation can be derived from the Neher and McGrath equations. In this case the earth thermal resistance is separated into "self" and "mutual" heating terms. Each term is multiplied by the appropriate losses. By combining these terms the formulation of Equations 1 and 2 of the paper can be derived. The process is repeated for each cable to determine which is the hottest. The ampacity should be the same as derived from the Neher and McGrath equations enumerated above although a small difference will occur if the temperatures of all cables are assumed to be the same.

In applying the authors' equations 1 and 2 it should be realized that the earth thermal resistance is not actually a function of heat. Although the equivalent mutual heating factor of equation 2 is a function of heat flows the earth thermal resistance is assumed constant. The temperature rise produced by each heat source is directly proportional to the heat generation. This is significant since in practice the earth thermal resistivity may be variable with heat flow but the equations are not meant to account for this phenomenon.

The next topic in this paper is a so-called improved method for calculating Rsd, the thermal resistance from cable to duct or pipe.

The Neher & McGrath paper describes in some detail how data from heat transfer tests on cables installed in ducts or steel pipes were used to compute the constants in Equations 41, 53 and 54. In these tests (references [2] and [3]) the cables were resting as they do in an actual installation in contact with the conduits. The tests covered various duct materials having different conductivity and a range of cable sizes and duct diameters were covered. The fill ratios ranged from about 0.2 to 0.9. (Higher fill ratios are impractical since cables must be pulled into ducts and a minimum clearance or % fill is normally recommended.)

The test results clearly showed that where gas in the intervening medium the conductivity of the duct material strongly influences the heat transfer as does the diameter of the cable. The alternate equations (8-12) in this paper do not fit available data for cables installed in ducts or pipes where air or compressed nitrogen is the intervening medium. Errors as large as -50% in Rsd and +12.5% in ampacity or +25% in temperature rise of conductor over ambient will occur using these equations for non metallic ducts. Errors on the conservative side of as much as +70 to +100% in Rsd occur for cables in steel pipe with air or high pressure nitrogen as the medium.

Table A-1 summarizes the results of tests for cables in fiber or Transite ducts and Table A-2 for cables in steel pipes and compares the measured thermal resistance with that computed by Equations 41 and 53 of Neher and McGrath and equations 8-12 of the Black and Sellers paper.

For the case of 3 cables in a pipe we have tabulated the total loss and computed Rsd for an equivalent single conductor cable so that the thermal resistance has units of degree C per total watts. Equations 8, 10 and 12 of the paper for thermal conductance have the factor  $nn'$  (number of conductors) in the numerator rather than the denominator but in this case  $nn' = 1$  so the effect of this error is nil. Application of equations as shown results in an additional error of  $1/n^2$  in Rsd for multiple conductors in a duct ( $1/9$  for 3 conductors per duct).

As can be seen from the table the Neher and McGrath equations fit the data quite well (within 20% for equation 41 and within 15% for equation 53).

The Black and Sellers method results in substantial deviations from the data (typically -20% to -50% for non-metallic ducts and +50% to +70% for cables in steel pipes).

The above discrepancies result at least in part since equations 8-12 are based on correlations for concentric cylinders rather than for cylinders which are thermally in contact.

Table B compares thermal resistance derived from tests on high pressure pipe cables with various dielectric liquids. These are compared with Equations 41 and 54 of Neher and McGrath and also with correlations derived from tests at the EPRI Waltz Mill test site conducted by Underground Systems, Inc. The Black and Sellers equations (with the  $nn'$  factor moved to the denominator) were evaluated for the same data and are also tabulated. How-

ever, the radiation term which is does not apply, was omitted.

The Neher and McGrath equations agree with the data for high viscosity mineral oil since this data was used to derive the equations. However the Neher and McGrath equations do not agree as well with the data for lower viscosity liquids. For these, either the Waltz Mill correlation or the equations of the present paper are more accurate.

For the case of cables installed in steel pipes filled with dielectric liquids, equations 8-12 agree quite well with available data. In this case the conductance through the liquid is large compared with that directly from cable to pipe which the equations ignore.

Typically the accuracy of the Neher and McGrath equations for *naturally* cooled pipe cables is sufficient since the temperature rise from cable to pipe is small compared with the total temperature rise from conductor to ambient. However the Waltz Mill tests were undertaken to determine more accurate formulations for the temperature rise of the cable to the fluid in *forced-cooled* mode where this temperature rise can be a significant parameter.

The third topic of the paper concerns heat conduction through earth of non-homogeneous conductivity.

The conformal mapping technique is not new. It is a standard method for solving two dimensional field problems. The method was applied to cable problems at least as far back as 1958 (L. Fink and J. Smerke, reference [4]). Other papers on the same subject appeared in the AIEE Transactions and in the IEE in England (Luoni [5]).

Finally the technique was the subject of a CIGRE Working Group report (reference [6]).

Properly applied, this technique is very useful. When the backfill thermal resistivity differs from that of the surrounding native earth, the position of cables within the backfill as well as the backfill shape can effect the resulting temperature rise and can be evaluated by this technique. These effects are not adequately treated by the Neher and McGrath paper, although for the majority of cases encountered in practice the errors involved are not significant compared to the errors involved in estimating the thermal resistivities.

#### References

1. Neher and McGrath, AIEE Transactions on Power Apparatus and Systems, October 1957.
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Table A-1  
Rsd Comparison for Cables in Conduit

1	Type	F	F	F	F	F	F	F	F	T	T	T
2	Ds	3.5	3.5	3.5	0.69	0.69	0.69	3.13	3.13	3.5	3.5	3.5
3	Dp	4.0	4.0	4.0	3.5	3.5	3.5	3.5	3.5	4.0	4.0	4.0
4	Ds/Dp	.875	.875	.875	.20	.20	.20	.89	.89	.875	.875	.875
5	$\delta$	0.3	0.3	0.3	1.4	1.4	1.4	0.2	0.2	0.3	0.3	0.3
6	Tm	56.6	69.8	69.9	57.5	67.1	78.1	50.8	51.9	56.8	69.5	64.3
7	Watts/ft	12.4	19.9	23.6	2.50	6.60	12.3	0.90	2.40	16.7	26.3	25.2
8	dT	16.6	24.3	25.2	15.1	34.2	56.1	1.60	3.80	17.1	24.4	25.5
9	Measured Rsd = 8 $\div$ 7	1.34	1.22	1.07	6.04	5.18	4.56	1.78	1.58	1.02	0.93	1.01
10	Eq. 41	1.23	1.14	1.14	4.80	4.60	4.38	1.41	1.41	1.01	0.93	0.96
11	10 $\div$ 9	0.92	0.93	1.07	0.80	0.89	0.96	0.80	0.89	0.98	1.01	0.95
12	Eq. 53	1.27	1.12	1.12	5.27	4.49	4.00	1.84	1.69	1.04	0.94	0.96
13	12 $\div$ 9	0.95	0.92	1.05	0.87	0.87	0.88	1.04	1.07	1.02	1.01	0.95
14	Black & Sellers Rsd	0.82	0.76	0.76	3.95	3.50	3.18	0.91	0.89	0.82	0.76	0.78
15	14 $\div$ 9	0.61	0.62	0.71	0.65	0.68	0.70	0.51	0.56	0.80	0.82	0.77

1. Type: F = Fiber, T = Transite duct, 1 cable with air at 1 atm.

2. Ds = o.d. of cable in inches

5.  $\delta$  = air gap dimension (Dp-Ds)/2

7. Measured heat flow in watts per ft

Source: References 2 and 3

3. Dp = id of conduit in inches

6. Tm = mean temperature of air space

8. dT = measured temperature rise of cable

surface over duct i.d.

Table A-2  
Rsd Comparison for Cables in Steel Pipe

1	Type	Steel-Air	HPGF	HPGF	HPGF	HPGF
2	Ds'	3.42	3.42	3.42	3.92	3.92
3	Dp	6.07	6.07	6.07	6.07	6.07
4	Ds'/Dp	.563	.563	.563	.646	.646
5	$\delta$	1.3	1.3	1.3	1.1	1.1
6	Tm	60.	51.	51.	35.	45.
7	Watts	23.4	28.6	28.9	6.8	15.9
8	dT	20.0	13.1	14.4	3.7	8.0
9	Measured Rsd-8 $\div$ 7	0.85	0.46	0.50	0.54	0.50
10	Eq. 41	0.88	0.53	0.53	0.49	0.48
11	10 $\div$ 9	1.03	1.15	1.06	0.91	0.95
12	Eq. 53	0.88	0.50	0.49	0.57	0.49
13	12 $\div$ 9	1.03	1.09	0.99	1.05	0.98
14	Black & Sellers Rsd	1.33	0.78	0.77	0.90	0.79
15	14 $\div$ 9	1.55	1.70	1.54	1.66	1.56

1. Type: Steel - air 3 cables in steel pipe with air at 1 atm.  
HPGF = 3 cables in steel pipe with Nitrogen at 14.7 atm
  2. Ds' =  $2.155 \times$  od of cable in inches
  3. Dp = id of conduit in inches
  5.  $\delta$  - air gap dimension  $(Dp-Ds)/2$
  6. Tm = mean temperature of air space
  7. Watts = measured heat flow in watts per ft
  8. dT = average measured temperature rise of cable surface over duct id
  9. Rsd = measured thermal resistance in  $^{\circ}\text{C}$  per total watts
- Source: Reference 2

Table B  
Rsd Comparison  
High Pressure Fluid Filled Pipe-Type Cable

1	Ds	2.78"	2.78"	2.78"	3.90"
2	Dp	8.125"	8.125"	10.25"	10.25"
3	Tm	40C	40C	40C	40C
4	$\Delta T$	5C	10C	10C	10C
5	Fluid Type	HVP	HVP	LVAB	LVAB
6	Eq. 41	.987	.987	.987	.792
7	Eq. 54	1.040	.908	.908	.734
8	Black & Sellers	.968	.843	.572	.452
9	Waltz Mill FC	.976	.822	.565	.388

1. Cable diameter (3 phases/pipe)
2. I.D. of pipe
3. Mean Fluid Temperature
4. Temperature rise of cable surface over pipe
5. HVP = High Viscosity Polybutene; LVAB low viscosity alkyl-benzene
6. Neher & McGrath Equation 41, Th Ohm-ft
7. Neher & McGrath Equation 54, Th Ohm-ft
8. Equations 8, 12 (revised with nn' in denominator), Th Ohm-ft
9. Waltz Mill Forced-Cooled Test data correlation, fully developed laminar flow, ref 7

Manuscript received March 13, 1995.

**Sally M. Sellers and W. Z. Black (Georgia Institute of Technology, Atlanta, GA)** We would like to thank the reviewers for their comments and questions concerning our paper. Their input will help clarify areas in the paper that were not thoroughly explained.

One common thread that appears in a majority of the reviewers comments involves our intent in writing this paper. Our primary goal was to present *refinements* to the Neher and McGrath paper. We had neither the resources nor the laboratory facilities to verify the proposed refinements to their thermal model. Obviously an experimental study of the extent necessary to verify, for example, the resistance of a series of duct banks, ranging from square to oblong would require a substantial investment of time, labor and capital. However, we are completely confident that the refinements to the Neher-McGrath model that are proposed in the paper are accurate, even though they have not been verified in laboratory tests. The refinements that we recommend are based on sound heat transfer principles and founded on heat transfer correlations that have been verified in laboratory settings. We are confident that when the accuracy of our refinements is checked, they will prove to be sound.

We also wish to reiterate our purpose in proposing the three refinements that are addressed in the paper. We were careful not to refer to these as corrections of errors or extensions to the equations that appeared in the model. In the title and throughout the text, we pointed out that we were proposing *refinements* to the Neher-McGrath method. We were also careful to label the curves that compared our proposed thermal resistance and ampacity values with those given in the Neher-McGrath paper as percent differences, not percent errors. Obviously the thermal model proposed by Neher and McGrath has withstood the test of time and it is, and will remain, the single most important paper dealing with the problem of ampacity in underground cables. Considering the assumptions stated in the paper and using the most up-to-date heat transfer correlations available in the 1950's, it represents a correct and error-free model. However, the sophistication of heat transfer knowledge has progressed significantly in the nearly forty years since this paper has been published. More refined convective heat transfer correlations covering a far broader range of geometrics and flow conditions are available today than existed in the 1950's. This factor alone should encourage a periodic reexamination of a thermal model such as the one proposed by Neher and McGrath. A reexamination in no way implies that the method is incorrect, simply that its accuracy could be enhanced through the incorporation of improved heat transfer correlations.

Furthermore, we feel the calculational procedure that was appropriate for the slide rule era of the 1950's should be periodically revisited in a rapidly evolving age of personal computers. Assumptions made in the original model to simplify the calculations and to avoid an iterative solution can now be relaxed. Using today's computers, the formidable task of repeating the calculations until the correct temperatures throughout the thermal circuit are known is no longer as time consuming as it was in the 1950's. Therefore, some of the assumptions that Neher and McGrath made in their formulation to simplify the calculations no longer need to be used and the model can be made more flexible and accurate.

Another aspect of the Neher-McGrath model should also be addressed. Their model is based on the electric analog of heat conduction through a series of solid media. In order for the analog method to successfully predict the ampacity of an underground cable installation, all thermal resistances in the circuit, the maximum conductor operating temperature, and the ambient soil temperature must be known. Furthermore, all sources of heat that are generated in the circuit such as those resulting from circulating currents in the shield, current in the conductor, and dielectric losses must be either known or expressed in terms of the conductor current. In this way, the thermal circuit can be reduced to a single equation in terms of the single unknown which is the current in the conductor. If any of the heat generation terms or the thermal resistance terms are temperature dependent, then the analog model requires that the local temperatures be approximated so that the terms can be accurately quantified and the model will retain its simple nature. This is the difficulty Neher and McGrath faced in several areas when they wrote their paper. They were well aware that the convective and radiative heat transfer resistances across air or liquid layers that exist in pipe-type cable installations are strongly temperature dependent. Therefore, they assumed the temperature differences and mean temperatures that existed in the air and fluid layers in the circuit. Furthermore the radiative resistances across gas layers are non-linear with temperature and Neher-McGrath chose to simplify the resistance by linearizing this term. Unless the surface temperatures of the two opposing, radiating surfaces are approximated, the analog method requires iteration on the various component temperatures until they converge to values that conserve energy for the entire installation. Obviously a mathematical model that requires repeated applications of the equations before converging to a correct set of temperatures is not a very popular approach unless a personal computer is used to solve the set of equations. Given today's sophisticated level of computing capabilities, it is no longer necessary to assume the local temperatures in the thermal network in order to estimate the temperature dependent resistance terms. Iterative algorithms are common-place and for this reason, it is no longer necessary to use some of the assumptions that were used by Neher and McGrath to mathematically simplify their model. This philosophy motivated us to suggest some of the refinements that we proposed in the paper.

In order to consolidate our comments to the questions posed by the reviewers, we will organize our responses in the same order as the three refinements that appear in the paper.

#### Mutual Heating Factor

The mutual heating parameter that we present in Eqn. 1 is equivalent to the way in which mutual heating is treated in the IEC Standards Publication 287 as pointed out by Dr. Anders. Since the factor F is a function of not only the geometry but also the rate at which heat is generated in every cable in the system, the temperature rise at any cable can only be determined once the ampacity in every cable is known. This fact forces an iterative solution in which the ampacities are assumed and the temperature rise resulting from each cable is calculated. The process continues until ampacity values converge to ones that provide the given conductor temperatures. If one assumes all cables produce the same amount of heat, the factor F becomes only a function of

geometry and an iterative procedure is not necessary. As we show in Fig. 2, in those situations for which the heat generated in the cables is not equal, the assumption of equal heat generation will cause ampacity errors less than 8 percent.

Our purpose in presenting a different form of the mutual heating parameter in Eqn. 2 of the paper was to point out how unequal heating influences the value of F. We are not implying that Neher and McGrath made an error in their consideration of mutual heating, only that unequal heating can be factored into the expression for F and unequal heating does not need to be handled by calculating interference temperatures as described in their paper. We feel that the simple modification to the mutual heating parameter as described in Eqn. 2 is less complex than the interference temperature technique described in the Neher and McGrath paper. The use of equations 48, 49, and 50 is not necessary if the expression for F is used to account for the effects of uneven heating. As we pointed out in the our paper, the expression for F given by Eqn. 2 reduces to Eqn. 46 in Neher-McGrath when all cables dissipate heat equally. The results in Figs. 1 and 2 were provided to show the percent differences in earth resistance and ampacity that will arise due to the assumption of equal heating when cables actually generate heat at different rates.

#### Resistance of Fluid Layers for Cables in Ducts and Pipes

The numerical error identified in Eqn. 5 by Dr. Morgan was discovered shortly after we submitted our paper for review and it has been corrected. This equation was provided in the paper for reference purposes only. It was not used to generate any of the curves in the paper.

Our comments in reference to Fig. 3 of the paper and our discussion of the thermal resistance of the fluid layer for cables in ducts and pipes attempted to point out the fact that Eqns. 41 and 41a in the Neher-McGrath paper were limited by several assumptions. Both of these equations do not express the thermal resistance in terms of the thickness of the fluid layer surrounding the cables. Also both of these equations do not include the influence of the emissivity of either the jacket or pipe material. Presumably both of these parameters are accounted for by the values of A, B and C (for Eqn. 41) or A' and B' (for Eqn. 41a). Unfortunately the values for these constants are provided only for a metallic conduit, only two types of duct (fiber and transite), and only three types of fluids (air at 1 atm, gas at 200 psi and oil). The user is unable to calculate the thermal resistance for other, more modern cable installations and more up-to-date duct materials. An equation such as Eqn. 10 that is expressed in terms of the emissivity of both the duct and jacket material and Eqn. 12 that is written in terms of the geometry and properties of the intervening fluid are more general and can be used for any emissivity, any cable and duct size, and any cable and duct material. Furthermore, they are not limited to any value for the mean temperature. Even though there are no experimental resistance measurements for cables in PVC ducts, for example, Eqns. 8-12 could be at least used to provide guidance for ampacity calculations. The user is therefore left with a calculational tool rather than forced to choose from a limited set of geometrics and system materials.

Mr. Fink points out that the expressions developed by Neher and McGrath for the air and oil thermal resistance were

validated in a series of experimental tests. In fact these experiments were used to determine the constants in Eqns. 41 and 41a. Chronologically the theoretical expression for the resistance of the convective, radiative and conductive heat transfer were formulated first and presented in the 1950 Buller and Neher paper. In order to derive these resistances, a number of assumptions were made, as pointed out in the paper. When the experimental work was completed, the theoretical expressions for the thermal resistances were corrected or adjusted to bring them into line with the experimental data. As Mr. Fink points out, some of the differences between the theoretical expressions and the experimental measurements may have resulted from the assumptions of concentric geometry and heat flow which is independent of azimuthal angle. The Buller and Neher expressions, as well as ours, are both based on these assumptions.

We feel that our expressions for the thermal resistances are more accurate than those provided in the paper by Buller and Neher, simply because they account for more the variables that are known to affect the heat transfer and they are not restricted by some of the limiting assumptions applied by Buller and Neher. We show a comparison between the two models later in this discussion to support this conclusion. Nonetheless, we recognize that our thermal resistance equations remain unverified by experimental measurements. We would welcome a series of tests that involved modern installation materials and practices. These tests are long overdue and would provide a valuable service to the power industry.

Regarding the value of  $T_a$  that we used when evaluating the thermal resistance of the fluid layer, we use the temperature equal to the inner surface of the duct or pipe. We determine this value by taking an energy balance on each layer of the installation geometry. In that way the temperature profile throughout the entire geometry is known and any quantity that is temperature dependent can be accurately evaluated. Therefore it is not necessary to assume any temperature to evaluate a temperature dependent thermal resistance. This process also results in a more general thermal model whose accuracy is improved because we do not have to assume any intermediate temperatures. The driving potential for heat transfer across the fluid layer is  $T_j - T_a$ , where  $T_a$  is the temperature of the inner surface of the duct, which is also equal to the air temperature in contact with the duct. In the paper we define  $\Delta T$  as the temperature difference between the outer surface of the cable and the inside surface of the duct.

Regarding the thermal resistance equations for cables in conduit, we provided equations that are similar to the ones presented by Buller and Neher in 1950. Like Buller and Neher's equations, they have not been adjusted to bring them into line with experimental data. If we had adjusted them, obviously they would have come closer to matching the available experimental data. Since our equations have not been adjusted to fit experimental data, it seems more appropriate to compare our equations with those of Buller and Neher. We have carried out a comparison between the two and the results are shown in Table C-1 (we will use similar organization as Ernst and Purnhagen in their tables so that values can be easily compared). For the eleven cases compared in Table C-1 the resistances predicted by Eqns. 8-12 give results that are in general closer to the experimental results than those given in

the paper by Buller and Neher. Also, we have compared our results as a percent difference from the experimental results and not as a ratio as calculated by Ernst and Purnhagen. The values for percentage differences are provided in Table C-1.

We have carefully examined the data in Table A-2 provided by Ernst and Purnhagen and we are unable to confirm their interpretation of the results from Eqns. 8-12. The values that we calculate for the resistance of the fluid layer in the pipe are given in Table C-2 along with the percent difference between these values and the measured values. The percentage differences that appear in Table C-2 are significantly less than portrayed by the ratio values in row 15 of Table A-2. We are uncertain of the exact values that Ernst and Purnhagen used for thermo-physical properties of the fluids so this may be the source of the differences. We have included the properties that we used in Table C-2 so there can be no misunderstanding as to how we calculated the resistance values. The values for the properties are from reference [B]. Our resistance values are slightly greater than those presented in the Buller-Neher paper. Finally we have verified all of the values that appear in Ernst and Purnhagen's Table B. For the conditions of oil in a pipe-type cable, Eqns. 8-12 predict fluid layer thermal resistances that are quite close to the Waltz Mill test data. The thermal resistance values predicted by Eqns. 41 and 41a in the Neher-McGrath paper and the correlations in the Buller and Neher paper show much larger differences. The fluid properties used to calculate the values in the table appear at the bottom of the table. These properties help explain an important point. Equations 41 and 41a are limited to the types of installations listed in Table VII of the Neher-McGrath paper. The constants in this table were determined after curve-fitting Eqn. 41 to experimental data. The experimental data was collected for a pipe-type cable that utilized an insulating oil with the properties shown in Table II. When the oil properties change to those of a high viscosity polybutene (HVP) or a low viscosity alkyl benzene (LVAB), the convective flow of the oil is changed and the ampacity can be significantly altered.

Since the constants in Eqns. 41 and 41a are matched to a pipe-type cable surrounded by a generic oil, it should not be surprising that the thermal resistances of an oil with different properties will produce large differences with measured values. These equations are being used in an application in which they were not intended to be used. On the other hand, Eqns. 8-12 have retained a general form that includes the properties of the oil and they would be expected to provide a better estimate for the oil layer when they are provided with accurate thermo-physical properties of the oil. In other words it should be expected that a general expression would be an improvement over an expression that is limited to a single fluid.

#### Resistance of Rectangular Duct Banks

Several questions were asked about the conformal mapping technique. We made no claims in the paper about this technique being new. It has obviously been used by others dating back many years. The conformal mapping technique that we used to calculate the thermal resistance of the duct bank does not assume the outer surface of the duct is an isothermal surface. Also it does not assume that the inner duct surface is uniformly heated. The thermal boundary conditions

that were assumed were an isothermal surface at the *inside* surface of the duct bank and an isothermal surface at the horizontal earth/air interface. These boundary conditions do not result in an isothermal duct bank surface as was assumed by Neher and McGrath. The details of the conformal mapping technique are given in reference 9 in the paper. They may also be found in an IEEE paper [D] that will be submitted for review in the next few months.

As pointed out in the paper, we have used both a finite element program (see Ref. 10 in the paper) as well as a conformal mapping model to evaluate the influence of non-square duct banks on the ampacity of cables. We ultimately selected the conformal mapping method for reasons of simplicity. We were concerned about having a relatively small ampacity program being burdened by a much larger and more complex finite element program. We chose to use the conformal mapping program for a wide variety of duct geometries and calculate the shape factors defined in Eqn. 16. The conformal mapping model calculates a ratio of the effective resistivity to the conduction shape factor. In doing so, the computational time is minimized and the model retains a reasonable degree of simplicity. Further, calculating an effective thermal resistance of the duct bank and earth allows us to keep the same form for the duct bank thermal resistance proposed by Neher and McGrath (see Eqn. 17). We have found that use of a finite element program to achieve the same goal requires far greater computational time with no improvement in accuracy.

Regarding Dr. Morgan's question about the influence of solar heating, the thermal model for ampacity assumes that the ambient soil temperature and the air/earth interface is maintained at a constant temperature. Rarely is the soil temperature constant. In most locations the surface temperature more closely follows seasonal variations in the air temperature, while the temperature variations at greater depths

are dampened and lag behind swings in the surface temperature. For any period in which the temperature of the surface of the earth is warmer than the subsurface temperature, the ampacity should be reduced if it is calculated on the basis of the ambient earth temperature at the cable depth. If the surface temperature is less than the soil temperature at the cable depth, the ampacity could be increased. Determining the precise amount the ampacity could be increased or decreased depends upon the soil resistivity and the temperature distribution in the soil and is not a simple matter to quantify. From a practical standpoint, the soil conditions in the immediate vicinity of the cable have a far greater influence on cable ampacity than the conditions of the soil near the earth/air interface. It is probably far more important to be concerned about the local variations in the soil resistivity at the cable depth than variations of the soil temperature. After all, the effect of variations in the local soil resistivity caused by local changes in soil composition or moisture content, for example, are probably far greater on the ampacity than the influence of variations in local soil temperature with depth. Therefore a thorough knowledge of the thermal resistivity of the soil at the cable depth along the entire route of the circuit is more critical to a calculation of the ampacity than a knowledge of the temperature profile of the soil as a function of depth.

#### References

- [A] Greebler, Paul, and Guy F. Barnett, "Heat Transfer Study on Power Cable Ducts and Duct Assemblies," *AIEE Transactions on Power Apparatus and Systems*, Vol. 69, pp. 357-367, 1950.
- [B] CRC Handbook of Tables for Applied Engineering Science, 2nd. ed., Ray E. Bolz and George L. Tuve, 1973.

TABLE C-1  
COMPARISON OF THERMAL RESISTANCE FOR CABLE IN CONDUIT

1	Type	F	F	F	F	F	F	F	F	T	T	T
2	$D_s$	3.5	3.5	3.5	0.69	0.69	0.69	3.13	3.13	3.5	3.5	3.5
3	$D_a$	4.0	4.0	4.0	3.5	3.5	3.5	3.5	3.5	4.0	4.0	4.0
4	$T_m$	56.6	69.75	69.9	57.5	67.1	78.1	50.8	51.9	56.75	69.5	64.25
5	$\Delta T$	16.6	24.3	25.2	15.1	34.2	56.1	1.60	3.80	17.1	24.4	25.5
6	W/ft	12.4	19.9	23.6	2.50	6.60	12.3	0.90	2.40	16.7	26.3	25.2
7	$R_{exp.}$	1.34	1.22	1.07	6.04	5.18	4.56	1.78	1.58	1.02	0.93	1.01
8	$R_a$	0.78	0.73	0.73	3.99	3.56	3.25	0.87	0.85	0.78	0.73	0.75
9	% diff.	<b>-41.4</b>	<b>-40.2</b>	<b>-31.8</b>	<b>-34.0</b>	<b>-31.4</b>	<b>-28.7</b>	<b>-51.1</b>	<b>-46.6</b>	<b>-23.5</b>	<b>-21.2</b>	<b>-26.3</b>
10	$R_a$	0.86	0.80	0.80	3.94	3.50	3.18	0.95	0.92	0.86	0.80	0.81
11	% diff.	<b>-36.0</b>	<b>-34.7</b>	<b>-25.5</b>	<b>-34.7</b>	<b>-32.5</b>	<b>-30.3</b>	<b>-46.6</b>	<b>-41.8</b>	<b>-16.5</b>	<b>-14.0</b>	<b>-19.6</b>

References [6] and [A]

- Type: F = fiber, T = transit duct
- $D_s$  = O. D. of cable in inches, 1 cable with air at 1 atm
- $D_a$  = I. D. of conduit in inches
- experimental thermal resistance (5+6)
- thermal resistance as predicted by Buller and Neher (eqn. 5)
- percent difference in thermal resistance between lines 7 and 8
- thermal resistance as predicted by eqns. 8 - 12
- percent difference in thermal resistance between lines 7 and 10

Radiative Properties:

- emissivity of jacket: 0.95  
emissivity of inner surface of duct: 0.80

- [C] Purnhagen, D. W., "Designers Handbook for Forced - Cooled High Pressure Oil Pipe Type Cable Systems," EPRI EL - 3624, Palo Alto, CA, July 1984.
- [D] Hartley, J. G., and Sandra J. Woods, "A Method for Determining the Effective Thermal Resistivity of Rectangular Duct Banks," to be submitted for review to *IEEE Transactions on Power Apparatus and Systems*, 1995.

TABLE C-2  
COMPARISON OF THERMAL RESISTANCE FOR  
CABLES IN STEEL PIPE

1	Type	steel /air	HPGF	HPGF	HPGF	HPGF
2	$D_s'$	3.42	3.42	3.42	3.92	3.92
3	$D_a$	6.07	6.07	6.07	6.07	6.07
4	$T_m$	52.0	51.0	51.0	35.0	45.0
5	$\Delta T$	20.0	13.1	14.4	3.7	8.0
6	W/ft	23.4	28.6	28.9	6.8	15.9
7	$R_{exp.}$	0.855	0.458	0.498	0.544	0.503
8	$R_a$	1.037	0.641	0.633	0.724	0.630
9	% diff.	21.4	39.9	26.9	33.0	25.2
10	$R_a$	1.167	0.665	0.656	0.763	0.665
11	% diff.	36.6	45.2	31.6	40.3	32.1

Reference [6]

- Type: steel/air = 3 cables in steel pipe with air at 1 atm  
HPGF = 3 cables in steel pipe with  $N_2$  at 14.6 atm
- $D_s' = 2.155 \times$  O. D. of cable in inches
- $D_a =$  I. D. of conduit in inches
- experimental thermal resistance (5+6)
- thermal resistance as predicted by Buller and Neher (eqn. 5)
- percent difference in thermal resistance between lines 7 and 8
- thermal resistance as predicted by eqns. 8 - 12
- percent difference in thermal resistance between lines 7 and 10

Properties used in eqns. 8 - 12 for Nitrogen Gas [B]:

conductivity: 0.000261 W/cm<sup>2</sup>C  
kinematic viscosity: 0.01074 cm<sup>2</sup>/s  
Prandtl number: 0.7327

Radiative Properties:

emissivity of jacket: 0.95  
emissivity of inner surface of pipe: 0.70

TABLE C-3  
COMPARISON OF THERMAL RESISTANCE FOR  
HIGH-PRESSURE FLUID FILLED PIPE-TYPE CABLES

1	Type	HVP	HVP	LVAB	LVAB
2	$D_s$	2.78	2.78	2.78	3.90
3	$D_a$	8.125	8.125	10.25	10.25
4	$T_m$	40.0	40.0	40.0	40.0
5	$\Delta T$	5.0	10.0	10.0	10.0
6	$R_{exp.}$	0.976	0.822	0.565	0.388
7	$R_a$	0.956	0.841	0.882	0.642
8	% diff.	-2.01	2.35	56.2	65.4
9	$R_a$	0.968	0.843	0.572	0.452
10	% diff.	-0.85	2.59	1.31	16.5

Reference [C]

- $D_s =$  O. D. of cable in inches (3 phases/pipe)
- $D_a =$  I. D. of conduit in inches
- experimental thermal resistance
- thermal resistance as predicted by Buller and Neher (eqn. 2 in [6])
- percent difference in thermal resistance between lines 6 and 7
- thermal resistance as predicted by eqns. 8 - 12
- percent difference in thermal resistance between lines 6 and 9

Properties used in eqns. 8 - 12 [C]:

HVP = High Viscosity Polybutene  
conductivity: 0.001092 W/cm<sup>2</sup>C  
kinematic viscosity: 1.2181 cm<sup>2</sup>/s  
Prandtl number: 1915.0  
coefficient of thermal expansion: 0.00075 1/<sup>o</sup>C

LVAB = Low Viscosity Alkylbenzene  
conductivity: 0.001102 W/cm<sup>2</sup>C  
kinematic viscosity: 0.1928 cm<sup>2</sup>/s  
Prandtl number: 276.8  
coefficient of thermal expansion: 0.00075 1/<sup>o</sup>C

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